List of Qualifying Exam Topics

Group Theory

- groups, normal subgroups, and group homomorphisms
  - ☐ cosets and quotient groups
  - ☐ Lagrange's Theorem
  - ☐ the Isomorphism Theorems
  - ☐ the center of a group
  - ☐ cyclic (sub)groups
- group actions
  - ☐ kernels and stabilizers
  - ☐ the Orbit-Stabilizer Theorem
  - ☐ the Class Equation
- direct products of groups
- finite (abelian) groups
  - ☐ the Fundamental Theorem of Finite Abelian Groups
  - ☐ invariant factors and elementary divisors of a group
  - ☐ Cauchy's Theorem for finite groups
  - ☐ Sylow's Theorems
- finitely generated (abelian) groups
  - ☐ (sub)groups generated by a set of elements
  - ☐ the commutator subgroup
  - ☐ the Fundamental Theorem of Finitely Generated Abelian Groups
  - ☐ the Smith Normal Form
- permutation groups
  - ☐ the symmetric group on n letters
  - ☐ the alternating group on n letters
  - ☐ Cayley's Theorem
  - ☐ Euler's Theorem
- semidirect products (*Note:* this is not explicitly needed, but it is useful!)

Ring Theory

- rings, ideals, and ring homomorphisms
  - ☐ the Subring Test
  - ☐ units and zero divisors of a ring
  - ☐ the hierarchy of commutative rings
  - ☐ principal / finitely generated ideals
  - ☐ prime / maximal ideals
  - ☐ the Jacobson radical of a ring
  - ☐ local rings
  - ☐ the localization of a ring at a multiplicatively closed subset
  - ☐ quotient rings
  - ☐ the Isomorphism Theorems
  - ☐ the Chinese Remainder Theorem
  - ☐ extension and contraction of ideals
  - ☐ Zorn's Lemma (*Note:* this is not explicitly needed, but it is useful!)
  - ☐ Oka families of ideals (*Note:* this is not explicitly needed, but it is useful!)
  - ☐ Noetherian rings (*Note:* this is not explicitly needed, but it is useful!)
- the hierarchy of integral domains
  - ☐ Euclidean domains
  - ☐ principal ideal domains (PIDs)
  - ☐ unique factorization domains (UFDs)
  - ☐ greatest common factor (GCD) domains
  - ☐ irreducible and primitive elements
  - ☐ the Gaussian integers
  - ☐ the field of fractions of an integral domain
- polynomial rings over UFDs
  - ☐ the Factor Theorem
  - ☐ irreducibility
  - ☐ Gauss's (Little) Lemma
  - ☐ Eisenstein's Criterion for Irreducibility

Field Theory

- field extensions
  - ☐ the minimal polynomial of an algebraic element
  - ☐ the degree of a (finite) field extension
  - ☐ expressing the inverse of an element in a field extension
  - ☐ the Conjugation Isomorphism Theorem
  - ☐ the splitting field of a polynomial
  - ☐ separability of a polynomial / field extension
  - ☐ the nth cyclotomic polynomial
- Galois Theory (*Note:* this is not explicitly needed, but it is useful!)

Linear Algebra

- linear transformations and matrices
  - ☐ the Rank-Nullity Theorem
  - ☐ the Isomorphism Theorems
  - ☐ the dual space and the annihilator of a vector (sub)space
  - ☐ the characteristic / minimal polynomial
  - ☐ the Cayley-Hamilton Theorem
  - ☐ eigenvalues and eigenvectors
  - ☐ T-invariant / cyclic subspaces
  - ☐ the Primary Decomposition Theorem
  - ☐ the Jordan Canonical Form
  - ☐ the Rational Canonical Form
  - ☐ the Smith Normal Form
- inner product spaces
  - ☐ the orthogonal complement of a vector space
  - ☐ the Gram-Schmidt Process
  - ☐ the Spectral Theorem for Symmetric Matrices
  - ☐ normal / Hermitian / unitary operators
  - ☐ the Spectral Theorem for Normal Operators